NO-ARGUMENTS

Denials, Refutations, Negations and the Constitution of Arguments

Shahid Rahman[1]

(Université de Lille, UMR: 8163, STL)

Abstract:

L. Horn's book *The Natural History of Negation* (Chicago UP, 1989) set both a landmark on the study of negation and a challenge. The challenge is to find some general way to understand what negation is. In fact, while for logicians and philosophers negation is a sentence building operator standardly understood as the reversal of truth and falsity for linguists negation involves a complex network of phenomena that go beyond the notion of sentence operator. Now, since the arrival and development of new notions of logic, different kinds of negation were formulated – sometimes understood as standing in competition: negations with metalogical features (involving the rejection of non-monotony), negation as different to the fact that a proof of a given proposition is still lacking (involving the rejection of the validity of third excluded), negation as modality (involving the rejection of non-contradiction and *ex-falso sequitur quodlibet*) and so on. In fact the paper ends with the proposal for a general theory of meaning embracing all these different logical constants.

My aim is to show how the role of negation in argumentation yields an approach general enough to capture the meaning of all this fauna of negative logical constants. The point is thus to show the fruitfulness of an argumentative approach to the study of a notoriously resilient logical constant. Though, it is about how argumentative perspectives can contribute to logical issues, the idea behind is certainly that if the logic is already developed in an argumentative frame; the further task to develop abstract structures to study "real argumentation" should be made easier at least more naturally (more on this perhaps in the discussion)

1.1 Acts of Denial and Antagonistic Arguments

The main claim is that different types of negative logical constants can be seen as passing from an act, then to a metalogical operator and at last to a sentence operator.

The notion of act of denial I am aiming at does not include refusal of proposals, however it shares with such refusals the feature that the result is not the negation of a sentence.

If I make the proposal "If you come to the cinema with me, I will give you a kiss" and you answer "No thank you very much!" you are in fact turning down a proposal and this act is not equivalent to

[1] shahid.rahman@univ-lille3.fr.

the negation of the conditional sentence involved in the proposal: turning down my proposal is not equivalent at all with “You come to the cinema and I do not give you a kiss”.

Moreover a proposal is not a claim! And that is why turning down a proposal does not amount to a negative claim.

More precisely, I will develop the idea that some basic acts of denial are constitutive of an important kind of argument – I will call those acts of denials *antagonistic denials* and the correspondent arguments *antagonistic arguments*:

The idea behind antagonistic arguments is that instead of asking for the reasons for a specific claim (why/because) one confronts the discussion partner with what appear to be the logical consequences of his own point of view (- in the recent AI- literature these type of arguments are called *Socratic*) [2]. or more generally one makes apparent to the antagonist that his point of view lacks of *formal (or material) grounding*.[3]

Let me avoid misunderstandings, I do not mean at all that the only kind of arguments are antagonistic arguments, but only that they provide a fruitful frame to study different kinds of negative logical constants. Similarly my aims are not to deal with all kinds of negations such as turning down proposals. My sole aim, once more is to show the fruitfulness of the argumentative approach even in the context of the meaning of a logical constant. Moreover, I do think that the study of turning down proposal deserves a thorough study, but this is not what I am doing here.

There is no antagonistic argument without a main claim, there is no main claim of an antagonistic argument without the corresponding act of denial of an assertion. In other words antagonistic arguments occur when there is an initial assertion involving a sentence φ that becomes a claim (the claim that φ can be grounded) by the very act of denying it and this interaction triggers the mutual interchange of defences and challenges.

The process by which the initial assertion becomes a claim requires the willingness of the player, called the *Proponent*, (for short ***P***) to accept the commitment to ground the sentence involved in the act of denial of the contender (the *Opponent*) under some specific initial conditions or unconditionally – the conditions specify a context under which the claim might be said to hold. The

---

[2] Caminada. A formal account of Socratic-style argumentation. Journal of Applied Logic 6(1):109-132 (2008)
[3] The lack of formal grounding does not in general amount to contradiction. The conditonal *If it rains, the streets will be wet* has by its own no formal grounding but it is not a contradiction .

Opponent, the player that performs the relevant act of denial, is linked to the task of building at least one counterexample to the main claim of the contender, called *thesis*. Let me call *weak-refutation* the result of building a counterexample to the thesis.

It is important to point out already that though the Proponent is always the defender of the thesis and the Opponent the challenger of thesis, during the argumentation process the Opponent might have to adopt the role of defending a sub-claim and vice versa for the Proponent.

Antagonistic denials are in principle of metalogical nature and by nature relational. With this I mean:

1. Such kind of denial is not part of a sentence, as standard negation is, but is in fact an act in relation to the act of assertion of a sentence – an act that makes of the initial assertion a claim and that is not expressed by any sentence in the object language.
2. Since the result of this basic denial is not part of a sentence (but the whole process of the attempt to build a weak refutation of the thesis) it can not be challenged as a whole – though some or each of the parts of the attempt to build a weak refutation that involve object language sentences can be challenged). In fact, antagonistic denials have an interactive component that in the case of logical proof has a strategic feature: namely, to confront the proponent with his own logical consequences – this is the nearest we come to strategic aspects of the game of the "lions part", though le me repeat that turning down of proposals do not constitute an antagonistic argument, since a proposal is not a claim. .[4]

Thus, we have the following picture:

Metalogical Denial

Opponent		*Proponent*
[act of denial]		Thesis (main claim)
……………..		……………..
(Attempt to build a weak Refutation)		(Attempt to ground the thesis)
…………….		……………...
Successful weak refutation	OR	Successful grounding

[4] In other words, the denegation of a sentence is not a logical constant (that build sentences). Hintikka expresses this fact by saying, and rightly so, that the this denial (that he calls contradictory negation) does not yield a rule to play *in* the game– since playing rules involve logical constants.

Compare with

Proposals

Abelard: *Proposal*: A

Turning down the proposal]: Eloise: No thank you

…………….. ……………..

Arguments

1) Eloise: why do you propose A?
Abelard: claim….

2) Abelard why not?
Eloise: claim

1.1.1 Metalogical Negation-operators

If those actions that yield weak refutations are built in the object language by a player as a component of an argument, that is, if they are expressed as an operator that is part of a sentence, say $\mathscr{F}\varphi$, they constitute a claim with an embedded denial in relation to φ.

Indeed by stating $\mathscr{F}\varphi$ the player claims that he will be able to develop a weak refutation (i.e. will be able to build a counterexample) to the sentence under the scope of the operator. Since this kind of objet-language denial is an operator that builds a sentence it can be challenged. Thus, the defence of such an operator commits to an act of denial of φ, that triggers a sub-argument with φ as a thesis:

the player who (weakly) asserted $\mathscr{F}\varphi$ will overtake in the sub-argument the task of developing a weak refutation of φ (in some contexts it makes sense to allow that the initial conditions under which $\mathscr{F}\varphi$ has been stated change in the sub-argument).[5]

Object-Language Denial

Utterance: *Y*: $\mathscr{F}\varphi$, *C1…C* (negative weak claim of φ: *Y* claims that under conditions *C1…Cn* there is (at least) a weak refutation of φ)

***X's*-challenge** of $\mathscr{F}$

***Y'*defence:** act of denial of φ: Y opens a sub-argument and challenges *X* to ground φ under the conditions *C1…Cn*

---------------------SUB-ARGUMENT------------

X: φ, *Y: C1…Cn*

(Attempt to ground the sub-claim φ under the conditions *C1…Cn*) to the sub-claim φ)

***X* builds a weak refutation to $\mathscr{F}\varphi$ iff he manages to ground φ in the Sub-argument**

These operators are object language negation-operators since:

(i) they build sentences that might constitute a component of an argument
(ii) the rules that fix their meaning are player independent. This is the main feature of the meaning of an operator in argumentative contexts. The meaning of an operator that builds a sentence must be independent of the player that utters the sentence. If not the sentence means something else for the two players and then no argument is possible.

[5] This kind of negation-operator is linked to negation by default (in non-monotonic reasoning) and has also been deployed to render the semantics of connexive negation.

(iii) Their meaning amounts to a switch – metalogical denials do not amount to a switch. Now this switch is rather complicated, it is not only a switch between defender a challenger but it might also involve the switch between the role of building a weak refutation. In fact; if a claim $\mathscr{F}\varphi$ is the thesis, the development of the play; will commit the Proponent to develop a weak refutation to the sub-argument-claim of the Opponent. This kind of switch is particularly important in the context of *formal arguments* (I will come back to this later on).

Such kind of operators have some metalogical features since the switch triggered involves the development of a sub-argument constituted by a sub-claim and a corresponding act of denial

1.1.2 Denial and the Constitution of Material and Formal Arguments

If we would like to avoid to have the result that an atomic sentence is true by the only reason that the player X stated it – that is, if we want to find a way out of contentious dialogues, then there are two possible ways:

- either we accept some principle of grounding external to the argument itself (and thus external to the interaction of the players)

  or

- we look for a player principle of grounding that is internal to the argument and dependent on the interaction of the players.

The first conception leads to material arguments the second to formal ones

If we are willing to accept something like *material truth*, then we can think that the grounds upon which the atomic formulae depend are facts of the world and a grounded atomic formulae is a way to say that it is true. However, something more general might be thought too, such as true in virtue of some player-independent ground. This constitutes the notion of *material arguments*

Material Arguments Only atomic grounded sentences may be uttered.

In material arguments the argumentation process constituted by the Opponent's attempt to build a weak refutation of the thesis and the Proponent's attempt to ground it) triggered by a denial, depends at the very last upon a principle that places the grounds (or refutation) of the thesis outside the argument.

Formal arguments are those where one of the players must play without knowing what the antagonist's justifications of the atomic formulae are. Thus, according to this view, the passage to formal arguments relates to the switch to some kind of games with incomplete information and the only strategy available to ground an atomic sentence is to use a copy-cat strategy that amounts to the following[6]:

- Me, the player X, can utter the atomic sentence *p* because you (player Y) uttered it before

Now, if the ultimate grounds of a thesis are atomic formulae and if this is implemented by a strategy as the one mentioned above the rule that constitutes formal arguments must be necessarily asymmetric. Indeed, if both contenders use such a strategy no atomic sentence can ever be uttered. Thus, we implement the so called formal rule by designing one player, called the *proponent*, whose utterances of atomic formulae are, at least, at the start of the dialogue restricted by this rule.

Formal Arguments
P may not utter an atomic sentence unless **O** uttered it first. Atomic sentences can not be challenged

Notice that we are not speaking of the logical notion of validity. An argument can be formal and even be won without being valid. Validity requires the extra notion of winning strategy defined for a given logical language.

Validity
A formula is valid in a certain logical system for argumentation iff **P** has a formal winning strategy for this formula..

[6] This has been worked out by Helge Rückert in a talk at the worshop Proofs and Dialogues, Tübingen, Wilehm-Schickard Institut für Informatik, 25-27; February 2011/.

I will not delve here into this notion but let me stress the following point. Helge Rückert (2011) pointed out, and rightly so, that the formal rule triggers a novel notion of validity.[7]. Validity, is not being understood as being true in every model, but as *having a winning strategy independently of any model* or more generally independently of any *material* grounding claim (such as truth or justification). The copy-cat strategy implicit in the formal rule is not copy cat ofgroundings but copy-cat of declarative utterances involving atomic formulae.

1.2 Negation-Operators

1.2.1 Dual Negation

There is a very basic negation-operator that we might call *dual negation* and that in its very general form relates to a switch to a parallel argumentative context, called its *dual.*[8] The dual of a dual drives us back to the original context. This kind of retrieval operation is called *involution*. For the sake of simplicity I will assume that there are only two argumentative contexts, the initial one (dialogical context 0) and its dual, the context 0* - though it is possible to set a more general frame where each dialogical context has its own dual.

The challenge to a dual-negation-operator commits the challenger to switch to a parallel argumentation-context where he becomes the defender of the positive dual of the challenged negation.

Dual negation is sentence building operator that can be subject of a denial-act and as such it might build assertions and claims. Moreover, differently to the metalogical negation-operator; there is no sub-argument with a new claim. More precisely; while in the sub-argument there might be a switch between the task to building a weak refutation, this will not happen in parallel contexts: the opponent will always be linked to the task of building a weak refutation independently of the argumentation-context in which the debate is taking place. That is why dual negation is not metalogical.

---

[7] Talk at the worshop Proofs and Dialogues, Tübingen, Wilehm-Schickard Institut für Informatik, 25-27; February 2011.

[8] Dual negation, is ubiquous in the literature: it is at the base of negation of the logic of first degree entailment (Routley's star-operator (Routley [1972], Anderson/Belnap [1975]), M. Dunn's relational many valued logic (Dunn [1976]), Hintikka's IF-negation (Hintikka [1996]) and the linear negation (Girard ??, Blass ??) . For a dialogical analysis see Rahman 2011.

Dual Negation

Utterance: *Y*: ~φ

**-------------------------PARALLEL-ARGUMENTATION-CONTEXT*------------------------
----------- *X's*-challenge** of ~: φ*

X utters φ at the dual context ***)**

Y: There is, strictly speaking no defence for *Y* , but a possible reply: that is, to launch a counterattack on φ at the dual context or switch back to the initial context and develop a counterattack there

Remark: **=initial context. This assumption adds the principle of double negation.

Picture 3

This negation is certainly not the standard negation. Indee, this negation does neither support the validity of third excluded nor of non-contradiction. - the point is that we will never have in the same argumentative context ~φ and its challenge φ*. However; the logic is not trivial, that is from a contradiction not everything follows. Furthermore, double negation holds since the assumption that the dual argumentation-context of a dual-argumentation context yields the initial context. Dual argumentation contexts represent a special simple case of modalisation. That is a set of arguments governed by a relation. So it is a case of going from one parallel argument to the other.

If we would like to avoid jumping from context to context, one can avoid the assumption of dual contexts and understand dual negation as a switch of choices in the same context and not as a switch of challenger and defender. That is, we understand here the dual negation as triggering a switch from a negated conjunction (disjunction) to a disjunction (conjunction) with negative disjuncts (conjuncts). As we will see later on, the switch between challenger and defender on the same context is in fact characteristic of the conditional.

1.3 Negation as a Conditional: Standard and not that Standard Negation-Operators

To obtain the core of the standard negation-operators, we simply drop the sign of argumentative context of the description above:

The Standard (Conditional) Negation-operators

Utterance: *Y*: ¬φ

***X's*-challenge** of ¬: φ

Y:

There is, strictly speaking no defence for *Y* , but a possible reply:

that is, to launch a counterattack on φ

Picture 4

In fact this kind of operators consider the negation to be a special case of a conditional, where there is no defence.

Indeed the core meaning of a conditional in this setting is:

The Conditional

Utterance: *Y*: φ →ψ

***X's*-challenge**: φ

Y'*s defence*: ψ

Picture 5

- Notice that the conditional triggers a switch of the challenger and defender roles in relation to the head of the conditional and that this switch happens in the same context in which the defence (the tail of the conditional) must be uttered.

Since in the negation there is no defence, the meaning of the negation-operator is identified as a switch of defender-challenger roles. However, on my view this stems from the understanding of negation as a conditional.

The idea behind that the utterance of $\neg\varphi$ amounts to uttering the conditional that establishes that:

if φ is the case, then some absurd sentence will be also the case.

($\varphi\rightarrow\bot$, the arrow is the conditional and "$\bot$" an arbitrary absurd sentence)

The sign "$\bot$", called *bottom*, stands in our setting to the result of the process of launching a counterattack.

In the preceding lines I talked about negation-operator in plural. The point is related to the distinction between two levels of meaning, namely, *local meaning* and *global meaning*.[9] Several negation-operators might share the same local meaning but differ in its local meaning.

The local meaning of an operator is given by some specific rules (called *particle rules*) that state the *local semantics*: what is at stake is only the challenge and the defence corresponding to the utterance of a given logical constant, rather than the whole context where the logical constant is embedded – picture 4 describes the local meaning of conditional-negation-operators).

Global meaning is included in the *structural rules*. Structural rules determine the general course of an antagonistic argument, whereas the particle rules regulate those moves (or utterances) that are challenges (to the moves of a rival) and those moves that are defences (to the challenges). Take the case of chess, there are those rules that determine how, say, the queen moves (corresponds to the particle rules) and those that determine who starts, that the moves are alternative, who wins and so on.

[9] This distinction stemms from the dialogical approach to logic. The main original papers are collected in Lorenzen/Lorenz 1978. A detailed account of recent developments can be found in Felscher 1985, Keiff 2004, Keiff 2007, Rahman 2009, Rahman/Keiff 2004, Rahman/Clerbout/Keiff 2009, Keiff 2009, Fiutek/Rückert/Rahman 2010, Rahman/Tulenheimo 2006, Rückert 2001, Rückert 2007. For a textbook presentation (in French), see Fontaine/Redmond 2008.

Global meaning is that part of the structural rules that determine the meaning of a sentence where a particle occurs as a main operator in the course of an argument. Take once more the the case of chess, one of the component of the global meaning is that once a move has been performed it cannot be withdrawn. Let me stress that global meaning is about the meaning *in the context of the development of a game* of a given logical constant that determines the logical form of a (possible complex) proposition. It is crucial to notice that while the dialogical approach considers the level of global meaning essential it does not prescribe which global rules should be fixed. Different global rules will determine different games: finite games, infinite games, alternating games, games where the switches between different parelle arguments are structured by a relation, and so on. [10]

Let us introduce the following different global meaning rules for negation-operators, that share all the local meaning o described in picture 4.

Classical rule In any move, each player may challenge a (complex) sentence uttered by his partner or he may defend himself against any challenge (including those challenges that have already been defended once).

This rule, as I will explain below, defines *the classical negation-operator* that yields the validity of double negation, third excluded and non-contradiction.

Intuitionistic Rule

[10] Let me stress that global meaning is about the meaning *in the context of the development of a game* of a given logical constant that determines the logical form of a (possible complex) proposition. Global rules include some choices on possibly delaying tactics. For short the players can choose how many times they might challenge or defend a move and this choice might be optimal or not to achieve their aims. However; from the point of view of the of meaning mentioned above there is no fixed number: this is dependent on the game and the players. From the point of view of **proving** there is indeed a fixed number – at least for classical and intuitionistic propositional logic – for classical logic the players must be allowed to repeat at least once their defences. This is related to the difference between play and strategic level discussed in the appendix. Thus, delays are possible and sometimes even necessary if the aim is to prove the logical validity for formula. There is some other sense of possible delay that amounts to the introduction of some other parallel arguments. Technically this amounts to the use of cut (very roughly it amounts to the transitivity of the consequence relation). If we would like the assure that the rules of the corresponding game fix meaning, then cut-elimination must be provable.

In any move, each player may challenge a (complex) sentence uttered by his partner or he may defend himself against the *last challenge* that has not yet been defended.

This rule defines intuitionistic negation-operator that yields the non-validity of one sense of double negation and third excluded – non-contradiction holds however.

Minimal-Logic Rule
In any move, each player may challenge a (complex) sentence uttered by his partner or he may defend himself against the *last challenge* that has not yet been defended, with the following proviso:

The Proponent can challenge a negative literal (the negation of an atomic sentence) iff the Opponent challenged *any other negative literal* before

This rule defines the negation-operator of minimal that yields the non-validity of one sense of double negation and third excluded and ex falso sequitur quodlibet– non-contradiction holds however

Paraconsistent Rule
In any move, each player may challenge a (complex) sentence uttered by his partner or he may defend himself against any challenge (including those challenges that have already been defended once), with the following proviso:

The Proponent can challenge a negative literal (the negation of an atomic sentence) iff the Opponent challenged *the same negative literal* before

This rule defines the negation-operator of one particular case of paraconsistent logic that yields the non validity of non-contradiction, one sense of double negation and and ex falso sequitur quodlibet– third excluded holds however.

Antagonistic arguments are triggered by acts of denials, internalized acts of denial are operators that might trigger a re-destribution of both the role of challenger and defender and the role of the player who has the duty to build a weak refutation. This re-distribution is performed at the level of sub-arguments – that is sections of an argument that might contain some or all of the information of the outmost upper section. It is important to point out that the link between sub-argument and its upper section is not only defined as a relation between sets of sentences but a relation resulting from the act of double re-distribution mentioned above.

According to my view dual negation is about switch between the roles of challenger and defender in dual contexts and this constitutes the rock bottom of a negation-operator.

Dual negation has some of the features of an internalized metalogical act of denial and can also be viewed as a kind of modal operator. Indeed,

- it is modal in the sense that it triggers a switch between parallel arguments but not of the formal rule (it does not redistribute the duty to build a weak refutation)

- it has still some of the features of an internalized act of denial since it triggers a switch: of the roles of challenger and defender (*pure* modal operators do not trigger a switch of players while switching between arguments). What makes of a negation a negation is the kind of switch that does not amount to taking information from one argument to the other – like modal operators do.

What about conditionals? Well, conditional-operators are (partial) challenger and defender switches in the *same argumentative context* – in a conditional the switch between defender and challenger happens only in relation to the head of the conditional (a total switch amounts to a conditional with a negation in its tail) in the same context and is not triggered by the switch between parallel ones. Perhaps conditional and negation are different special cases of the very notion of the duality between defender and challenger, a notion of duality that constitutes one of the most basic elements of an antagonistic argument. If we understand negation as a conditional, then the re-distribution is performed in the same argumentative context.
At this point we should either start again and talk more about the conditional or stop. I take the second option … for the moment.

Annex: A brief introduction to first-order dialogical games

By Nicolas Clerbout

Dialogical logic developed by Paul Lorenzen and Kuno Lorenz, was the result of a solution to some of the problems that arouse in Lorenzen' *Operative Logik* (1955) (Cf. Lorenz 2001). We can not discuss here thoroughly the passage from the operative to the dialogical approach, though as pointed out by Peter Schroeder-Heister, the insights of Operative logic had lasting consequences in the literature on proof-theory and still deserve attention nowadays (Schröder-Heister 2008). Moreover, the notion of *harmony* formulated by the antirealists and particularly by Dag Prawitz has been influenced by Lorenzen's notions of *admissibility*; *eliminability* and *inversion*. However, on my view, the dialogical tradition is rather a rupture than a continuation of the operative project and it might be confusing to start by linking conceptually both projects together.

Dialogical Logic was suggested at the end of the 1950s by Paul Lorenzen and then worked out by Kuno Lorenz.[11] Inspired by Wittgenstein's *meaning as use* the basic idea of the dialogical approach to logic is that the meaning of the logical constants is given by the norms or rules for their use. This feature of its underlying semantics quite often motivated the dialogical approach to be understood as a *pragmatist* semantics.[12]

The point is that those rules that fix meaning may be of more than one type, and that they determine the kind of reconstruction of an argumentative and/or linguistic practice that a certain kind of language games called dialogues provide. As mentioned above the dialogical approach to logic is not a logic but a semantic rule-based framework where different logics could developed, combined or compared. However, for the sake of simplicity and exemplification I will introduce only to the dialogical version of classical and intuitionist logics.

In a dialogue two parties argue about a thesis respecting certain fixed rules. The player that states the thesis is called Proponent (**P**), his rival, who puts into question the thesis is called Opponent

[11] The main original papers are collected in Lorenzen/Lorenz 1978. A detailed account of recent developments can be found in Felscher 1985, Keiff 2004a and 2004b, Keiff 2007, Rahman 2009, Rahman/Keiff 2004, Rahman/Clerbout/Keiff 2009, Keiff 2009, Clerbout 2011, Fiutek/Rückert/Rahman 2010, Rahman/Tulenheimo 2009, Rückert 2001, Rückert 2007. For a textbook presentation Redmond/Fontaine 2011.

[12] Quite often it has said that dialogical logics has a *pragmatic* approach to meaning. I concede that the terminology might be misleading and induce one to think that the theory of meaning involved in dialogic is not semantics at all. Helge Rückert proposes the more appropriate formulation *pragmatistische Semantik* (*pragmatist semantics*).

(**O**). In its original form, dialogues were designed in such a way that each of the plays end after a finite number of moves with one player winning, while the other loses. Actions or moves in a dialogue are often understood as *utterances* (Cf. Rahman/Rückert 2001, 111 and Rückert 2001, chapter 1.2) or as speech-acts (Cf. Keiff 2007). The point is that the rules of the dialogue do not operate on expressions or sentences isolated from the act of uttering them (Tulenheimo 2010). The rules are divided into particle rules or rules for logical constants (*Partikelregeln*) and structural rules (*Rahmenregeln*). The structural rules determine the general course of a dialogue game, whereas the particle rules regulate those moves (or utterances) that are requests (to the moves of a rival) and those moves that are answers (to the requests).

Crucial for the dialogical approach are the following points

1. The distinction between local (rules for logical constants) and global meaning (included in the structural rules)
2. The player independence of local meaning
3. The distinction between the play level (local winning or winning of a play) and the strategic level (existence of a winning strategy).
4. A notion of validity that amounts to winning strategy *independently of any model* instead of winning strategy for every model.
5. The notion of winning in a *formal play* instead of winning strategy in a model.

Let L be a first-order language built as usual upon the propositional connectives, the quantifiers, a denumerable set of individual variables, a denumerable set of individual constants and a denumerable set of predicate symbols (each with a fixed arity).

We extend the language L with two labels **O** and **P**, standing for the players of the game, and the question mark \`?'. When the identity of the player does not matter, we use variables **X** or **Y** (with **X**≠**Y**). A *move* is an expression of the form \`**X**-*e*', where *e* is either a formula φ of L or the form \`?[φ_1,..., φ_n]'.

We now present the rules of dialogical games. There are two distinct kinds of rules named particle (or local) rules and structural rules. We start with the particle rules.

Previous move	X-φ∧ψ	X-φ∨ψ	X-φ→ψ	X-¬φ

Challenge	Y–?[φ] or Y–?[ψ]	Y–?[φ,ψ]	Y–φ	Y–φ
Defence	X–φ resp. X–ψ	X–φ or X–ψ	X–ψ	- -

Previous move	X–∀xφ	X–∃xφ
Challenge	Y–?[φ(a/x)]	Y–?[φ(a_1/x),…,φ(a_n/x)]
Defence	X–φ(a/x)	X–φ(a_i/x) with 1≤ i ≤n

In this table, the a_is are individual constants and φ(a_i/x) denotes the formula obtained by replacing every occurrence of x in φ by a_i. When a move consists in a question of the form `?[$φ_1$,...,$φ_n$]', the other player chooses one formula among $φ_1$,...,$φ_n$ and plays it. We can thus distinguish between conjunction and disjunction on the one hand, and universal and existential quantification on the other hand, in terms of which player has a choice. In the cases of conjunction and universal quantification, the challenger chooses which formula he asks for. Conversely, in the cases of disjunction and existential quantification, the defender is the one who can choose between various formulas. Notice that there is no defence in the particle rule for negation.

Particle rules provide an abstract description of how the game can proceed locally: they specify the way a formula can be challenged and defended according to its main logical constant. In this way we say that these rules govern the local level of meaning. Strictly speaking, the expressions occurring in the table above are not actual moves because they feature formulas schemata and the players are not specified. Moreover, these rules are indifferent to any particular situations that might occur during the game. For these reasons we say that the description provided by the particle rules is abstract.

Since the players' identities are not specified in these rules, we say that particle rules are symmetric: that is, the rules are the same for the two players. The fact that the local meaning is symmetric (in this sense) is one of the biggest strengths of the dialogical approach to meaning. In particular it is the reason why the dialogical approach is immune to a wide range of trivializing connectives such as Prior's *tonk* (Cfr. Rahman et al. 2009; and Rahman 2010).

The expressions occurring in particle rules are all move schematas. The words "challenge" and "defence" are convenient to name certain moves according to their relationship with other moves. Such relationships can be precisely defined in the following way. Let Σ be a sequence of moves. The function p_Σ assigns a position to each move in Σ, starting with 0. The function F_Σ assigns a pair $[m,Z]$ to certain moves N in Σ, where m denotes a position smaller than $p_\Sigma(N)$ and Z is either C or D, standing respectively for "challenge" and "defence". That is, the function F_Σ keeps track of the relations of challenge and defence as they are given by the particle rules. Consider for example the following sequence Σ:

$$\mathbf{P}\text{-}\varphi\wedge\psi\ ,\ \mathbf{P}\text{-}\chi\vee\psi,\ \mathbf{O}\text{-}?[\varphi],\ \mathbf{P}\text{-}\varphi$$

In this sequence we have for example $p_\Sigma(\mathbf{P}\text{-}\chi\vee\psi)=1$

A *play* (or dialogue) is a legal sequence of moves, i.e., a sequence of moves which observes the game rules. The rules of the second kind that we mentioned, the structural rules, give the precise conditions under which a given sentence is a play. The *dialogical game* for phi, written $\mathbb{D}(\varphi)$, is the set of all plays with φ as the thesis (see the Starting rule below). The structural rules are the following:

SR0 (Starting rule) Let φ be a complex formula of $\mathbb{L}$. For every $\pi\in\mathbb{D}(\varphi)$ we have:

- $p_\pi(\mathbf{P}\text{-}\varphi)=0$,
- $p_\pi(\mathbf{O}\text{-n:=}i)=1$,
- $p_\pi(\mathbf{P}\text{-m:=}j)=2$

In other words, any play π in $\mathbb{D}(\varphi)$ starts with **P**-φ. We call φ the *thesis* of the play and of the dialogical game. After that, the Opponent and the Proponent successively choose a positive integer called *repetition rank*. The role of these integers is to ensure that every play ends after finitely many moves, in a way specified by the next structural rule.

SR1 (Classical game-playing rule)

- Let $\pi\in\mathbb{D}(\varphi)$. For every M in π with $p_\pi(M)>2$ we have $F_\pi(M)=[m',Z]$ with $m'<p_\pi(M)$ and $Z\in\{C,D\}$
- Let r be the repetition rank of player **X** and $\pi\in\mathbb{D}(\varphi)$ such that
  - the last member of π is a **Y** move,

- M_0 is a **Y** move of position m_0 in π,
- $M_1,...,M_n$ are **X** moves in π such that $F_\pi(M_1)=...=F_\pi(M_n)=[m_0,Z]$.

Consider the sequence[13] $\pi' = \pi * N$ where N is an **X** move such that $F_{\pi'}(N)=[m_0,Z]$. We have $\pi' \in \mathbb{D}(\varphi)$ only if $n < r$.

The first part of the rule states that every move after the choice of repetition ranks is either a challenge or a defence. The second part ensures finiteness of plays by setting the player's repetition rank as the maximum number of times he can challenge or defend against a given move of the other player.

SR2 (Formal rule) Let ψ be an atomic formula, N be the move **P**- ψ and M be the move **O**-ψ . A sequence π of moves is a play only if we have: if $N \in \pi$ then $M \in \pi$ and $p_\pi(M) < p_\pi(N)$.

That is, the Proponent can play an atomic formula only if the Opponent played it previously. The formal rule is one of the characteristic features of the dialogical approach: other game-based approaches do not have it.

A play is called *terminal* when it cannot be extended by further moves in compliance with the rules. We say it is **X** terminal when the last move in the play is an **X** move.

SR3 (Winning rule) Player **X** wins the play π only if it is **X** terminal.

Consider for example the following sequences of moves:

P-Qa∧Qb, **O**-n:=1, **P**-m:=6, **O**-?[Qa], **P**-Qa

P-Qa→Qa, **O**-n:=1, **P**-m:=12, **O**-Qa, **P**-Qa

The first one is not a play because it contravenes the Formal rule: with his last move, the Proponent plays an atomic sentence although the Opponent did not play it beforehand. By contrast, the second sequence is a play in $\mathbb{D}$(**P**-Qa→Qa). We often use a convenient table notation for plays. For example, we can write the play above as follows:

13 We use $\pi * N$ to denote the sequence obtained by adding move N to the play π.

	O			P	
				Qa→Qa	0
1	n:=1			m:=12	2
3	Qa	(0)		Qa	4

The numbers in the external columns are the positions of the moves in the play. When a move is a challenge, the position of the challenged move is indicated in the internal columns, as with move 3 in this example. Notice that such tables carry the information given by the functions *p* and *F* in addition to represent the play itself.

However, when we want to consider several plays together – or example when building a strategy - such tables are not that perspicuous. So we do not use them to deal with dialogical games for which we prefer another perspective. The *extensive form* of the dialogical game D(φ) is simply the tree representation of it, also often called the game-tree. More precisely, the extensive form E_{φ} of D(φ) is the tree (T,l,S) such that:

i) Every node t in T is labelled with a move occurring in D(φ)

ii) l: $T\rightarrow$ N

iii) $S\subseteq T^2$ with:

- There is a unique t_0 (the root) in T such that $l(t_0)=0$, and t_0 is labelled with the thesis of the game.
- For every $t\neq t_0$ there is a unique t' such that $t'St$.
- For every t and t' in T, if tSt' then $l(t')=l(t)+1$.
- Given a play π in D(φ) such that $p_{\pi}(M')=p_{\pi}(M)+1$ and t, t' respectively labelled with M and M', then tSt'.

Many metalogical results concerning dialogical games are obtained by considering them by leaving the level of rules and plays and moving to the level of strategies. Among these results, significant ones are given in terms of the existence of winning strategies for a player. We now define these notions and give examples of results.

A *strategy* for Player **X** in $\mathrm{D}(\varphi)$ is a function which assigns an **X** move *M* to every non terminal play π with a **Y** move as last member such that extending π with *M* results in a play. An **X** strategy is *winning* if playing according to it leads to **X**'s victory no matter how **Y** plays.

A strategy can be considered from the viewpoint of extensive forms: the extensive form of an **X** strategy σ in $\mathrm{D}(\varphi)$ is the tree-fragment $\mathrm{E}_{\varphi,\sigma}=(T_\sigma, l_\sigma, S_\sigma)$ of E_φ such that:

i) The root of $\mathrm{E}_{\varphi,\sigma}$ is the root of E_φ.

ii) Given a node *t* in E_φ labelled with an **X** move, we have that $tS_\sigma t'$ whenever tSt'.

iii) Given a node *t* in E_φ labelled with a **Y** move and with at least one t' such that tSt', then there is a unique $\sigma(t)$ in T_σ where $tS_\sigma\sigma(t)$ and $\sigma(t)$ is labelled with the **X** move prescribed by σ.

Here are some examples of results which pertain to the level of strategies.[14]

- Winning **P** strategies and leaves. *Let* w *be a winning **P** strategy in* $\mathrm{D}(\varphi)$. *Then every leaf in* $\mathrm{E}_{\varphi,\mathrm{w}}$ *is labelled with a **P** signed atomic sentence.*
- Determinacy. *There is a winning **X** strategy in* $\mathrm{D}(\varphi)$ *if and only if there is no winning **Y** strategy in* $\mathrm{D}(\varphi)$.
- Soundness/Completeness of Tableaux. *Consider first-order tableaux and first-order dialogical games. There is a tableau proof for* φ *if and only if there is a winning **P** strategy in* $\mathrm{D}(\varphi)$.

By soundness and completeness of the tableau method with respect to model-theoretical semantics, it follows that existence of a winning **P** strategy coincides with validity: *There is a winning **P** strategy in* $\mathrm{D}(\varphi)$ *if and only if* φ *is valid.*

Examples of extensive forms.

Extensive forms of dialogical games and of strategies are infinitely generated trees (trees with infinitely many branches). Thus it is not possible to actually write them down. But an illustration remains helpful, so we add Figures 1 and 2 below.

Figure 1 partially represents the extensive form of the dialogical game for the formula $\forall x(Qx\rightarrow Qx)$. Every play in this game is represented as a branch in the extensive form: we have given an example with the leftmost branch which represents one of the simplest and shortest plays

14 These results are proven, together with others, in Clerbout(2012).

in the game. The root of the extensive form is labelled with the thesis. After that, the Opponent has infinitely many possible choices for her repetition rank: this is represented by the root having infinitely many immediate successors in the extensive form. The same goes for the Proponent's repetition rank, and every time a player is to choose an individual constant.

Figure 2 partially represents the extensive form of a strategy for the Proponent in this game. It is a fragment of the tree of Figure 1 where each node labelled with an **O** move has at most one successor. We do not keep track of all the possible choices for **P** any more: every time the Proponent has a choice in the game, the strategy selects exactly one of the possible moves. But since all the possible ways for the Opponent to play must be taken into account by a strategy, the other ramifications are kept. In our example, the strategy prescribes to choose the same repetition rank as the Opponent. Of course there are infinitely many other strategies available for **P**.

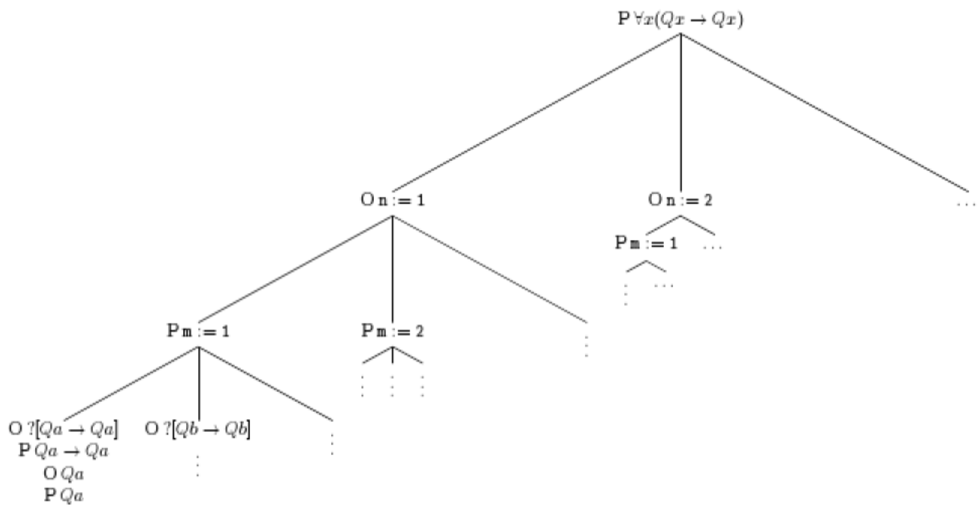

Figure 1.

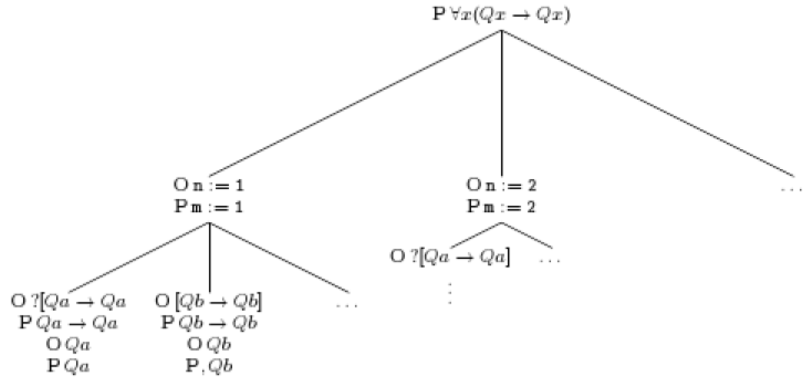
P ∀x(Qx → Qx)
O n := 1
P m := 1
O n := 2
P m := 2
...
O ?[Qa → Qa]
...
O ?[Qa → Qa
P Qa → Qa
O Qa
P Qa
O [Qb → Qb]
P Qb → Qb
O Qb
P, Qb
...

Figure 2.

References

CLERBOUT, N. (2011). Dialogical Games for First-Order Logic and Tableaux'. Paper presented at the *14e Congrès de Logique, Méthodologie et Philosophie des Sciences*. 19–26 juillet, Nancy (France).

CLERBOUT, N. / KEIFF, L. / RAHMAN, S. (2007): Dialogues and Natural Deduction. In G. Primiero and S. Rahman (ed.) *Acts of Knowledge, History, Philosophy, Logic*. G. Primiero (ed.), London : College Publications. chapter 4.

DUNN, M. (1976). Intuitive Semantics for First-Degree-Entailments and Coupled-Trees. *Philosophical Studies*, 29, 149-168.

FELSCHER, W.(1985). Dialogues as a foundation for intuitionistic logic. In D. Gabbay, D. and F. Guenthner, (editors), *Handbook of Philosophical Logic*, Volume 3, Dordrecht: Kluwer, 341–372.

REDMOND, J. / FONTAINE, M. (2011). *How to Play Dialogues. An Introduction to Dialogical Logic*. London : College Publications, London.

FIUTEK, V. / RÜCKERT, H. / RAHMAN, S. (2010) *A Dialogical Semantics for Bonanno's System of Belief Revision*. To appear in *Constructions* . P. Bour et alii (ed.), London : College Publications.

GENTZEN, G. (1935). Untersuchungen Ueber das Logische Schliessen. *Mathematische Zeitschrift* 39: 176–210.

HINTIKKA, J. (1999). *Inquiry as Inquiry: A Logic of Scientific Discovery*. Dordrecht: Kluwer.

HINTIKKA, J., HALONEN, I., AND MUTANEN, A. (1999). Interrogative Logic as a General Theory of Reasoning. In Hintikka 1999, pages 47–90.

KEIFF, L. (2004). Heuristique formelle et logiques modales non normales. *Philosophia Scientiae*, vol. 8-2, 39-59.

KEIFF, L. (2004). Introduction à la dialogique modale et hybride. *Philosophia Scientiae*, vol. 8-2, 89-105.

KEIFF, L. (2007). Approches dynamiques à l'argumentation formelle. PHD thesis, Lille: Université de Lille.

KEIFF, L. (2009). Dialogical Logic, Entry in the Stanford Encyclopaedia of Philosophy, 2009,http://plato.stanford.edu/entries/logic-dialogical/

KEIFF, L. / RAHMAN, S. (2010). La Dialectique entre logique et rhétorique. *Revue de Métaphysique et Morale*, Avril-June 2010, vol. 2, 149-178.

LORENZ, K. Basic objectives of dialogue logic in historical perspective. *Synthese*, vol. 127: 255–263.

LORENZEN, P. (1995) *Einführung in die operative Logik und Mathematik*. Berlin, Göttingen, Heidelberg: Springer.

LORENZEN P. / LORENZ K (1978). *Dialogische Logik*. Darmstadt: Wissenschaftliche Buchgesellschaft.

PRAWITZ, D (1979). Proofs and the meaning and completeness of the logical constants. In J. Hintikka, I. Niiniluoto, and E. Saarinen (editors), *Essays on Mathematical and Philosophical Logic,* Dordrecht: Reidel, 25–40.

PRIEST, G. (2001). *An Introduction to Non-Classical Logic*. Cambridge: Cambridge U. Press.

PRIEST, G. (2011). Realism, Antirealism and Consistency. In the present volume.

RAHMAN, S (1993*). Über Dialogue, Protologische Kategorien und andere Seltenheiten*. Frankfurt/Paris/ N. York: P. Lang.

RAHMAN, S. (2009)**.** A non normal logic for a wonderful world and more. In J. van Benthem et alia *The Age of Alternative Logics*, chez Dordrecht: Kluwer-Springer, 311-334.

RAHMAN, S. (2012)**.** Negation in the Logic of First Degree Entailment and Tonk. A Dialogical Study. In G. Primiero et alii (ed.). In (Anti)Realism. *The Realism-Realism Debate in the Age of Alternative Logics*. Dordrecht: Springer, 175-202.

RAHMAN, S. / CLERBOUT, N. / KEIFF, L. (2009). Dialogues and Natural Deduction. *Acts of Knowledge, History, Philosophy, Logic*. G. Primiero (ed.), London : College Publications, 301-336.

RAHMAN, S. / KEIFF, L. (2004). On how to be a dialogician. In D. Vanderveken (ed.), *Logic, Thought and Action,* Dordrecht: Kluwer, 359-408.

RAHMAN, S. / RÜCKERT, H. (2001)**.** *New Perspectives in Dialogical Logic*. Special volume of *Synthese*, vol. 127.

RAHMAN, S. / TULENHEIMO, T. (2009). From Games to Dialogues and Back: Towards a General Frame for Validity. In O. Majer, A-V. Pietarinen and T. Tulenheimo (editors), *Games: Unifying Logic, Language and Philosophy*, Part III, Dordrecht, Springer.

READ, S. (2008). Harmony and modality. In C. Dégremont, L. Keiff, and H. Rückert (editors), *Dialogues, Logics and Other Strange Things: Essays in Honour of Shahid Rahman*, London: College Publications, 2008, 285–303.

READ, S. (2010). General Elimination Harmony and the Meaning of the Logical Constants. *Journal of Philosophical Logic* 39: 557-576.

ROUTLEY, R. AND V. ROUTLEY. (1972). The Semantics of First Degree Entailment. *Noûs* 6: 335–95.

RÜCKERT, H. (2001). Why Dialogical Logic? In H. Wansing (ed.), *Essays on Non-Classical Logic*, N. Jersey, London …: World Scientific, 165-185.

RÜCKERT, H. (2007). Dialogues as a dynamic framework for logic. PHD-Thesis, Leyden, 2007. Online in: https://openaccess.leidenuniv.nl/dspace/bitstream/1887/12099/1/R%C3%BCckert_PhD_Dialogues_neu.pdf.

RÜCKERT, H. (2011). The Conception of Validity in Dialogical Logic. Talk at the workshop Proofs and Dialogues, Tübingen, organized by the Wilhelm-Schickard Institut für Informatik (Universität Tübingen), 25-27 February 2011.

SCHRÖDER-HEISTER, P. (2008). P. Lorenzen's operative justification of intuitionistic logic. In M. van Atten, P. Boldini, M. Bourdeau, G. Heinzmann (eds.), *One Hundred Years of Intuitionism* (1907-2007), Basel: Birkhäuser.

SMULLYAN; R. (1968). *First-Order Logic*. Heidelberg: Springer.

SUNDHOLM, B. G. (1983a). Constructions, proofs and the meaning of the logical constants. *Journal of Philosophical Logic*, Vol. 12, 151-72.

SUNDHOLM, B. G. (1983b). Systems of deduction', chapter I:2 in: Gabbay, D., and F. Guenthner, *Handbook of Philosophical Logic*, Vol. I, Reidel, Dordrecht.

SUNDHOLM, B. G. (2010). Proofs as Acts and Proofs as Objects: Some Questions for Dag Prawitz. *Theoria* 64 (2-3):187-216.

TULENHEIMO, T. (2010). On the dialogical approach to semantics. Talk at the Workshop Amsterdam/Lille: *Dialogues and Games: Historical Roots and Contemporary Models*, 8-9 February 2010, Lille. (Online inhttp://www.tulenheimo.webs.com/talks.html)